Perceptual quality evaluation of geometric distortions in images

Angela D’Angelo, Gloria Menegaz and Mauro Barni
University of Siena, Department of Information Engineering, Siena, Italy

ABSTRACT

Human perception of image distortions has been widely explored in recent years, however, research has not dealt with distortions due to geometric operations. In this paper, we present the results we obtained by means of psychovisual experiments aimed at evaluating the way the human visual system perceives geometric distortions in images. A mathematical model of the geometric distortions is first introduced, then the impact of the model parameters on the visibility of the distortion is measured by means of both objective metrics and subjective tests.

Keywords: geometric distortions, human visual system, perceptual quality.

1. INTRODUCTION

Despite the existence of several studies on human perception of image quality, research has not dealt with geometric distortions. This is an important problem in many applications such as digital watermarking in presence of de-synchronization attacks\(^1,2\). The use of digital watermarking in real applications, in fact, is impeded by the weakness of current available algorithms against signal processing manipulations leading to the de-synchronization of the watermark embedder and detector. For this reason the problem of watermarking under geometric attacks has received considerable attention throughout the recent years. Specifically it could be very useful to know the amount of geometric distortions that can be applied before the distorted image loses its commercial value or its meaning in order to develop an ad-hoc decoding algorithm and eventually obtain watermark synchronization through exhaustive search.\(^3\) Other applications concerned with geometric distortions are, for example, registration of biomedical images that usually requires local and nonlinear transformations,\(^4,5\) or collusion-secure fingerprinting techniques by random pre-warping.\(^6\) In these last methods the host signal is randomly warped prior to watermarking in such a way to prevent traitors from obtaining a high-quality copy through collusion. The random pre-warping must be strong enough to avoid that registration techniques can undo the warping and, in the meantime, it must guarantee the invisibility of the distortion. For this reason gathering information about the subset of perceptually admissible geometric distortions is a vital requirement.

In the above context the goal of this paper is twofold: first, to design a mathematical model to describe perceptually admissible geometric distortions; second, to measure the impact of the parameters of the model from a perceptual point of view. Specifically, the maximum admissible distortion that can be applied before the distortion becomes visible is measured leading to the definition of the perceptually admissible subset of the possible distortions that can be applied to the images.

The rest of this paper paper is organized as follow. In Section 2 we present the design and the hypothesis behind the proposed model. In Section 3 we present the implementation of the model. Section 4 regards the investigation of the perception of the geometrical distortions introduced by the model. In particular, both objective and subjective tests are performed to characterize the visibility of the corresponding artifacts. Finally, in Section 5 we derive the conclusions and propose some ideas for further research.

2. GEOMETRIC DISTORTIONS

In a general case a geometric distortion can be seen as a transformation of the position of the pixels in the image. It is possible to distinguish between global and local geometric distortions.

In this paper we decided to neglect the problem of global transformations because this class is small with respect to local transformations and for this reason it is easier to evaluate the amount of perceptually admissible
Local distortions, in fact, refer to transformations affecting in different ways the position of the pixels of the same image or affecting only part of the image. A general model which comes to mind to do this is a distortion according to which each pixel $Z(i, j)$ of the image is assigned a random displacement vector $\Delta(i, j) = (\Delta_h(i, j), \Delta_v(i, j))$, where $\Delta_h(i, j)$ and $\Delta_v(i, j)$ are i.i.d random variables uniformly distributed in the interval $[-\Delta_{max}, \Delta_{max}]$. The main problem in a so defined transformation is that it does not take into account the way the Human Visual System (HVS) perceives geometrical distortions. Our goal is to take into account the HVS to find a perceptually admissible subset of the possible distortions that can be applied to the image.

In the following subsections some models to treat geometric transformations are sketched. Each model is analyzed by means of visual inspection using the images in figure (1): one is the standard image Lena that is mostly smooth, the other one, the Duomo image, is much more structured.

### 2.1. Block-based Local Permutation (LP)

As explained above, a generic local distortion can be described, for example, by a permutation of the position of the pixels in the image. Of course this kind of distortion introduces an annoying degradation. A way to overcome this problem could be to fix a maximum displacement of the position of the pixels, i.e. to perform block-based local permutations.

This model consists in partitioning the $S \times S$ original image in $\frac{S}{Dim} \times \frac{S}{Dim}$ blocks and obtaining the distorted image by allowing random permutations within each block.

Let $Z(i, j)^{(k)}$ be a generic pixel of the distorted image $Z$ belonging to the $k$th block (with $k = \{1, 2, ..., (\frac{S}{Dim})^2\}$), then $Z(i, j)^{(k)} = Y_{\pi_k(i, j)^{(k)}}$, where $Y$ is the original image and $\pi_k(i, j)$ is a random permutation of the indices belonging to the $k$th block.

Increasing the value of $Dim$ allows to consider a larger number of transformations but, at the same time, affects the image quality leading to increasingly annoying artifacts.

Figure (2) shows the effect of this kind of distortion on the two sample images (considering $Dim = 5$). It is easy to deduce that the transformations generated with this model are not perceptually admissible.
2.2. Local Permutation with Cancelation and Duplication (LPCD)

In this model we add to the previous one the possibility of duplicating and canceling sample values so that it is also possible to model local expansions and shrinkings. Furthermore in this way we allow for a larger number of possible distortions.

Focusing on the 1D-case, let  \( y = \{y(1), y(2) \ldots y(n)\} \) be a generic signal and let  \( z = \{z(1), z(2) \ldots z(n)\} \) be the distorted version of  \( y \).

Let  \( z(i) \) be a generic element of  \( z \), the LPCD model states that  \( z(i) = y(i + \Delta_i) \) where  \( \Delta_i \) is a sequences of i.i.d random variables uniformly distributed in a predefined interval  \( I = [-\Delta_{\text{max}}, \Delta_{\text{max}}] \). For simplicity we assume that  \( \Delta_i \) can take only integer values in  \( I \).

To extend the model to the 2-D case, if  \( Z(i, j) \) is a generic pixel of the distorted image  \( Z \), we have

\[
Z(i, j) = Y(i + \Delta_h(i, j), j + \Delta_v(i, j)),
\]

where  \( Y \) is the original image and  \( \Delta_h(i, j) \) and  \( \Delta_v(i, j) \) are sequences of i.i.d integer random variables uniformly distributed in the interval  \( [-\Delta_{\text{max}}, \Delta_{\text{max}}] \).

Figure (3) shows the sample images distorted with the LPCD model with  \( \Delta_{\text{max}} = 2 \). It is possible to note that this distortion, even if it is not yet a perceptually admissible distortion, does not present the annoying block artifacts visible in figure (2), this is due to the overlapping of the windows of the possible displacements of neighboring pixels.

An important limit of the LPCD model is the lack of memory. The lack of memory is likely to be a problem from a perceptual point of view: with no constraints on the smoothness of the displacement field there is no guarantee that the set of LPCD distortions is perceptually admissible even by constraining  \( \Delta_{\text{max}} \) to be very small. This statement is confirmed by Figure (3).

2.3. Constrained LPCD (C-LPCD)

A way to overcome the limit of the lack of memory of the LPCD model, in order to obtain better results from a perceptual point of view, is to require that the sample order, in the 1D case, is preserved (thus introducing memory in the system). In practice, the displacement of each element  \( i \) of the distorted sequence  \( z \) is conditioned on the displacement of the element  \( i-1 \) of the same sequence. In formulas,  \( z(i) = y(i + \Delta_i) \) where  \( \Delta_i \) is a sequence of i.i.d integer random variables uniformly distributed in the interval  \( I = [\max(-\Delta_{\text{max}}, \Delta_{i-1} - 1), \Delta_{\text{max}}] \). Figure (4) illustrates an example of the behavior of the C-LPCD model in the 1D case, with  \( \Delta_{\text{max}} = 2 \). We know that  \( z(i) = y(i + \Delta_i) \), let us assume that  \( \Delta_i \) is chosen in the interval  \( I_i = [-2, 2] \) (the solid line box) and that  \( \Delta_i = 1 \), it means that  \( z(i) = y(i + 1) \) (as indicated by the arrow on the left). At the next step we know that  \( z(i+1) = y(i + 1 + \Delta_{i+1}) \) where  \( \Delta_{i+1} \), due to the position of the pixel  \( z(i) \), must be chosen in the interval...
Figure 3. Local Permutation with Cancelation and Duplication (LPCD) with $\Delta_{\text{max}} = 2$: (a) Lena; (b) Duomo.

$I_{i+1} = [0, 2]$ (the bold dotted line box). The interval $I_{i+1}$ (the bold dotted line box) is smaller than $I_i$ (the solid line box) because the position of the element $i+1$ cannot precede that of the element $i$. For example $\Delta_{(i+1)}$ could be equal to 2 yielding $z(i+1) = y(i+3)$ (as indicated by the arrow on the right).

Figure 4. Constrained LPCD with $\Delta_{\text{max}} = 2$ (1D case).

To extend the C-LPCD model to the 2-D case we let $Z(i, j)$ be equal to $Y(i + \Delta_h(i, j), j + \Delta_v(i, j))$, where $\Delta_h(i, j)$ is randomly chosen, to preserve the horizontal sample order, in:

$$I_h = \left[ \max \left( -\Delta_{\text{max}}, +\Delta_h(i - 1, j - 1), +\Delta_h(i - 1, j - 1) - 1 \right), \Delta_{\text{max}} \right],$$

and $\Delta_v(i, j)$ is randomly chosen, to preserve the vertical sample order, in:

$$I_v = \left[ \max \left( -\Delta_{\text{max}}, +\Delta_v(i - 1, j), +\Delta_v(i - 1, j) - 1 \right), \Delta_{\text{max}} \right].$$

In other words the horizontal and vertical displacement of the pixel $(i, j)$ are limited by the horizontal and vertical displacements of the pixels $(i-1, j)$, $(i, j-1)$ and $(i-1, j-1)$. Specifically, the lower bound of the intervals $I_h$ and $I_v$ of each pixel $(i, j)$ is determined by the positions of the pixels $(i-1, j)$, $(i, j-1)$ and $(i-1, j-1)$ while the upper bound depends only on $\Delta_{\text{max}}$.

Figure 5 shows the Duomo and the Lena images distorted with the C-LPCD model (considering $\Delta_{\text{max}} = 2$).

After a visual inspection conducted on the images we can deduce that constrained LPCD is more perceptually admissible model than the previous models.
2.4. Multiresolution Constrained LPCD (MC-LPCD)

To further improve the C-LPCD model, and to make the distortions less perceptible, we considered a last model, a Multiresolution version of C-LPCD (MC-LPCD). In this case the Constrained LPCD model is applied at different resolutions to obtain the global displacement field. A low resolution displacement field is first generated, then a full size displacement field is built by means of bilinear interpolation. The full resolution field is finally applied to the original image to produce the distorted image.

More specifically, MC-LPCD consists of two steps. In the first step C-LPCD is applied at a low level of resolution to obtain the displacement fields $\delta_h(i, j)$ and $\delta_v(i, j)$ with size $\frac{S}{L} \times \frac{S}{L}$ where $L$ is the level of resolution and $S$ is the size of the original image. The displacement fields $\delta_h$ and $\delta_v$ are resized to the original image size by means of bilinear interpolation to obtain the full resolution displacement fields $\Delta_h$ and $\Delta_v$. Note that in this phase the values of the displacement fields are not enlarged in order to make the distortions less perceptible and take advantage of the multiresolution model. For this reason new non-integer displacement values are introduced. The full resolution displacement fields $\Delta_h$ and $\Delta_v$ are then applied to $Y$ through a bilinear interpolation of the gray levels to find the final distorted image $Z$ (introducing in this way new gray levels in the images).

3. A MARKOV CHAIN INTERPRETATION OF C-LPCD MODEL

After a visual inspection conducted on a set of images distorted with the MC-LPCD and C-LPCD models we observed that rather surprisingly changing the value of $\Delta_{max}$ does not seem to change the perceptual quality of the images. This can be explained by resorting to the theory of Markov Chains.

Let us go back to the constrained LPCD model described in Section 2.3 and let us focus on the 1-D case, it is possible to design a Markov chain whose states correspond to the possible sizes of the interval $I = [\max(-\Delta_{max}, -\Delta_{i-1} - 1), \Delta_{max}]$, representing the set of elements of the original sequence among which the element of the distorted sequence is selected. By choosing for example $\Delta_{max} = 2$, the state space $S$ is $\{1, 2, 3, 4\}$ where the state $S_i$ corresponds to a size of $I$ equal to $i+1$, i.e. state 2 corresponds to the set $I = \{0, 1, 2\}$ of size 3, while state 4 corresponds to $I = \{-2, -1, 0, 1, 2\}$ of size 5 and it is the maximum size that $I$ can have (with $\Delta_{max} = 2$). The $4 \times 4$ transition matrix of this Markov Chain, representing all the possible one-step transition probabilities among the states of the chain, is:
and the corresponding graph is reported in figure (6). To exemplify the above concepts, let us assume to be in state 3 at a generic step \( i \), corresponding to \( I = \{ -1, 0, 1, 2 \} \). By looking at the graph, it means that, at step \( i + 1 \) we have one fourth of probability to be in state 1, one fourth to be in state 2 and the same probability to be in state 3 or 4.

\[
P = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{2}{5}
\end{bmatrix}
\]

where \( \Delta = \Delta_{\text{max}} \) and each element of the matrix \( p_{ij} \) is the transition probability of going from state \( i \) to state \( j \). From Markov chain theory\(^7\) we know that if a Markov chain is aperiodic and irreducible then there exists a unique limit distribution that can be found by solving the following system:

\[
\begin{align*}
\pi_j &= \sum_{i \in S} \pi_i p_{ij} \quad \forall j \in S \\
\sum_{j \in S} \pi_j &= 1
\end{align*}
\]

where \( \pi_j^{(n)} = P\{X_n = i\} \) is the probability to be in state \( i \) after \( n \) steps.

It is easy to show that in our case the chain is irreducible and aperiodic thus it is possible to find a unique limit distribution by solving the system in equation (2) that now is equivalent to:

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Figure 6. Graph of the Markov chain describing the C-LPCD model, for \( \Delta_{\text{max}} = 2 \).

In a more general case, given \( \Delta_{\text{max}} \), the maximum size of \( I \) is equal to \( N = 2\Delta_{\text{max}} + 1 \) and the transition matrix of size \( 2\Delta_{\text{max}} \times 2\Delta_{\text{max}} \) is:

\[
P = \begin{bmatrix}
\frac{1}{2} & \frac{1}{2} & 0 & \cdots & \cdots & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & \cdots & 0 \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{1}{2\Delta+1} & \frac{1}{2\Delta+1} & \frac{1}{2\Delta+1} & \cdots & \cdots & \frac{2}{2\Delta+1}
\end{bmatrix}
\]

(1)
After some algebraic manipulations it is possible to find the limit probability distribution of states as expressed in (3):

\[
\begin{align*}
    \pi_1 &= \sum_{i=1}^{2\Delta} \frac{1}{i+1} \\
    \pi_j &= \sum_{i=j-1}^{2\Delta} \frac{1}{i+1} \quad \text{with } j \text{ going from } 2 \text{ to } 2\Delta - 1 \\
    \pi_{2\Delta} &= \frac{1}{2\Delta+1} \sum_{i=1}^{\Delta} \pi_i + \pi_{2\Delta} \cdot \frac{2}{2\Delta+1} \\
    \sum_{j=1}^{2\Delta} \pi_j &= 1
\end{align*}
\]

Figure (7.a) shows the limit probability of the states (i.e., the probability to be in state \(i\) after \(n\) steps when \(n \to \infty\)) as a function of \(N\) (\(N = 2\Delta_{\text{max}} + 1\)). Interestingly, regardless of the value of \(N\) and therefore of \(\Delta_{\text{max}}\) states 1 and 2 are by far the most probable states (\(\pi_1 = \pi_2\) except when \(\Delta_{\text{max}} = 1\)). This is due to the second equation in (3) and explains why changing the value of \(\Delta_{\text{max}}\) does not change the perceptual quality of the image. In fact even by increasing the value of \(\Delta_{\text{max}}\) states 1 and 2 are the most probable states, but state 2 corresponds to the interval \(I = \{0, 1, 2\}\) of size 3 so even increasing the value of \(N\) the model tends to prefer only small displacements. Furthermore it is easy to verify that the states reach their limit probability distribution already for very small values of \(n\).

Of course, by increasing the value of \(N\) the limit probabilities of the states decreases because the number of possible states increases, in particular the minimum value of \(\pi_1\) is approximately equal to 0.3679, as was found by solving the following limit:

\[
\lim_{N \to \infty} \pi_1 = \lim_{N \to \infty} \frac{1}{2N-2} \sum_{k=0}^{2N-2} \frac{1}{k!} + \frac{2N+1}{(2N)!} = \frac{1}{e} \approx 0.3679
\]

Interestingly, by observing figure (7.a) it is possible to note that this limit is already reached for small values of \(N\). Since \(\pi_1 = \pi_2\), it is clear why changing the value of \(\Delta_{\text{max}}\) does not change the perceptual quality of the image.

To avoid this undesirable effect and to allow the model to generate a larger variety of displacement fields, we modified the Markov chain described by the graph in (7) by changing the transition probabilities among the states in order to give a major probability to those transitions that allow a larger interval \(I\). A way to do this is to assign the same probability (equal to \(\frac{1}{2\Delta+1}\)) to those transitions that cause a decrease of the size of \(I\), corresponding to the elements \(i, j\) with \(i = 1, \ldots, \Delta_{\text{max}}\) and \(j = 1, \ldots, i\) of the transition matrix, and to assign all the remaining probabilities, equal to \(1 - \sum_{j=1}^{i} p_{ij}\), to the transition corresponding to the element \(i, j\) with \(i = 1, \ldots, \Delta_{\text{max}}\) and \(j = i + 1\), i.e. those transitions whose effect is to enlarge the interval \(I\). The corresponding transition matrix becomes:
Figure (7.b) shows the limit probability distribution of states versus $\Delta_{\text{max}}$ of the new Markov chain founded in a numerical way.

By comparing figures (7.a) and (7.b) it is evident that with the modified Markov Chain is possible to obtain larger displacement fields because regardless of the value of $\Delta_{\text{max}}$ all the states have almost the same limit probabilities.

By observing the last figures it is clear that this model provides a better way to incorporate perceptual considerations within the model. In particular the image quality increases, from a perceptual point of view, if the MC-LPCD model is applied to a lower level of resolution but, in the meantime, the number of possible distortions decreases.

4. EXPERIMENTAL RESULTS

Given the good performance of the MC-LPCD model (the modified version presented in Section 3), we now present the experimental results obtained by means of both objective and subjective tests. To do so, let us observe that from a perceptual point of view MC-LPCD has a different behavior for different values of N and for different levels of resolution L.

The goal of the tests was to establish the sensitivity of the visual system to the geometric distortions introduced by the model as a function of the control parameters N and L. In this way we were able to identify the range of variation of the control parameters that do not affect image quality.
Figure 8. MC-LPCD applied at different resolution levels: (a) $L = 3$, (b) $L = 4$, (c) $L = 6$. 
For objective testing, we used the PSNR measurement and state-of-the-art metrics such as the Universal Quality Index,\textsuperscript{8} the SSIM-index\textsuperscript{9} and the RST based metric developed by I.Setyawan et al.\textsuperscript{10} It is worth mentioning that despite the great research effort, there is still a lack of objective visual quality metrics suitable for geometric distortions.

In the subjective test we applied the two alternatives forced choice (2AFC) paradigm: the users were asked to compare two images at time, the original image and the distorted image, and to indicate which one was the original.

The source image database used in both the tests included sixteen gray scale images, 512 × 512 pixel in size, and was derived from a set of source images that reflects adequate diversity in image contents. The images, in fact, included pictures of faces, houses, natural scenes and images without any specific object of interest. Some images have high activity, while some do not have much structures and are mostly smooth.\textsuperscript{11}

We chose to distort the source images through the MC-LPCD model using different distortion types obtained by changing the dimension of \( N \) (\( N = 5, N = 7, \) and \( N = 9 \)) and the level of resolution (\( L = 6, L = 5, L = 4, L = 3, L = 2 \)). Specifically we produced thirteen distorted versions for each image (all the possible combinations of \( N \) and \( L \) except \( N = 9 \ L = 2 \) that always generates a visible distortions and \( N = 9 \ L = 6 \) because the level of resolution is smaller than \( N \)), for a total of 208 images.

4.1. The objective test

Many image quality assessment algorithms have been shown to behave consistently when applied to certain kinds of distortions (e.g., JPEG compression), but the effectiveness of these metrics degrades when they are applied to a set of images distorted geometrically. However, just for completeness, we present the results we obtained by applying some of these metrics to the images used in the subjective test. For a given set of parameters, the comparison of the subjective test described in the following subsection with the objective quality measures will allow to establish the difficulty of objective metrics in predicting the perceived amount of geometrical distortions present in an image.

The results of the objective test are shown in figure (9). We do not show the results obtained with the Universal Quality Index because SSIM index it is an improvement of it.

![Figure 9](image)

\( \text{Figure 9.} \) Plot of objective metrics versus the level of resolution used in the MC-LPCD model: (a) Peak Signal to Noise Ratio; (b) Structural Similarity based IMage index.

We also applied the RST based metric developed by Setyawan\textsuperscript{10} but the results we obtained are not meaningful. In our case, in fact, this metric is not able to predict the visual quality of the images because it does not find a local RST or affine transform (neither in a small interval) approximating the geometric distortion introduced by the MC-LPCD model.
4.2. The two alternative forced-choice test

The subjective test we used is the two alternative forced-choice test (2AFC). Two stimuli are presented at each trial. One of these stimuli is the original image; the other one is a distorted version of it, and the observer is asked to select the original image. Procedures for such experiment have been designed by following the ITU-T Recommendation P.910, which suggests standard viewing conditions, criteria for the selection of observers and test material, assessment procedures, and data analysis methods.

The experiments were conducted by using the VP800 video card of the Cambridge Research Systems together with a high resolution digital monitor Mitsubishi DiamondPro 2070 with the external adaptor ViSaGe 71.02.00D2. To have a correct color representation a luminosity calibration was previously done through a videocamera ColorCAL.

The tests involved a panel of fifteen subjects, all naives with respect to image quality assessment methods and image impairments. Each subject was individually briefed about the goal of the experiment, and given a demonstration of the test. Subjects were shown images in a random order, the randomization was different for each subject. The test was performed in a dark room in free viewing conditions.

In order to analyze the results obtained with the subjective test an hypothesis test was conducted. This test tells us whether the subjective test, based on the number of sample points used, allows to make a statistically sound conclusion about the behavior of the MC-LPCD model. The test statistic on which the hypothesis test is based is a probability test: we tested the hypothesis $H_0$ that the probability $p = P(A)$ of an event $A$ equals a given constant $p_0$, using as data the number $k$ of successes of $A$ in $n$ trials. Specifically we tested the hypothesis that the probability $p$ that the users choose the original image in the 2AFC test (event $A$) is equal to $p_0 = \frac{1}{2}$ (it means that the original image and the distorted image are perceptually indistinguishable) using as observable data the number of times that the users decided for the original images in the 240 total comparisons (16 images $\times$ 15 users). The alternative hypothesis is that $P(A) < \frac{1}{2}$ and we use $\alpha = 0.05$ as significativity level.

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Table 1. Maximum admissible distortion that can be applied to the images before the distortion becomes visible using the MC-LPCD model.

With the hypothesis test we derived the maximum admissible distortion that can be applied before the distortion becomes visible using the MC-LPCD model. Specifically, the results of the hypothesis test are shown in table (1). For each level of resolution and for each typology of images, we found the maximum value of $N$ that can be used while keeping the distortion invisible (the hypothesis $p_0 = \frac{1}{2}$ is accepted). The empty boxes correspond to cases in which it was not possible to find an adequate value of $N$, corresponding to that level of resolution, resulting in an invisible distortion. By looking at the results we note that, as expected, in images that do not have much structures, like natural images, the introduced distortions are less visible. Furthermore the results in table (1) show the difficulty of the objective metrics in predicting the degradation introduced by the model in the images: for example the plot $N = 5$ in figure (9.a) shows a loss of quality going from $L = 6$ to $L = 5$ while the subjective test assures the invisibility of the distortion in both cases. In the same way the loss of about 5 dB in the PSRN plot between the configurations $N = 5$ $L = 6$ and $N = 7$ $L = 6$ is not meaningful according table (1).

5. CONCLUSION AND FUTURE WORKS

In this paper we have proposed a model to describe perceptually admissible distortions introduced by geometric transformations. The first part of the paper presented the design and the implementation of the proposed model, while the second part regarded the investigation of the perception of the distortions introduced by the model.
Specifically we established, through a two alternative forced choice test, the sensitivity of the human visual system to geometrical transformation as a function of the control parameters $L$ and $N$ in such a way to identify the range variation of these parameters that cause distortions that do not affect image quality.

The idea behind the model is the assumption that the displacement field of the pixel at location $(i, j)$ is conditioned, in order to prevent annoying artifacts, to those of the pixels at locations $(i - 1, j), (i, j - 1)$ and $(i - 1, j - 1)$. An improvement that should be addressed in future works is to extend this system considering for each pixel the displacement fields of a first order neighborhood system (or even a second order one) associated with it, thus obtaining a multidirectional model.

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