Compressed sensing techniques for hyperspectral image recovery

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ABSTRACT

Compressed Sensing (*CS*) theory is progressively gaining more interest over scientists of different fields. The reason is its potential to provide high resolution capture of physical signals from relatively few measurements, tipycally well below respect to the limit given by the Shannon/Nyquist sampling theorem. Sampling a signal with few measurements gives the big advantage of sampling and compressing *simultaneously* that signal.

One of the fields which could gain more benefits from CS theory is image compression: in the normal compression process, we have to turn a large digital data set into a smaller one, but in many applications could be useful to avoid the initial large data set to begin with, and to acquire and sampling at the same time. We apply the CS theory to optimize the capturing process of Hyperspectral Images, which are characterized by an huge amount of data with high spatial and spectral correlation and, hence, allows a compact (i.e., quasi-sparse) representation in a 3D domain.

The aim of the paper is twofold: (i) to investigate to sparseness degree S, i.e., the number of nonzero samples in the transform domain which are necessary to reconstruct the signal with satisfactory quality, i.e., with quality comparable to typical lossy compression schemes; (ii) to investigate the number of measurements M which are necessary to reconstruct the signal with satisfactory quality, whereas reconstruction is performed by means of l1-norm minimization and acquisition is performed by means of random matrices.

INTRODUCTION

The limited resources of a satellite and the need of preserving radio-band could have great benefits from an already compressed acquisition process. Existing signal acquisition schemes are based on Shannon‘s theorem, which requires the sampling frequency to be at least twice as large as the signal‘s maximum frequency. Compressive sampling (CS) is a new signal acquisition and processing paradigm invented in 2005 (see [4]), which revolutionizes Shannon‘s sampling by exploiting the notion that most natural signals, e.g. images, are highly correlated (see [12]). Correlation implies that there exists a domain in which the signal is sparse; for example, only a small fraction of the wavelet coefficients of natural images are significantly different from zero. CS investigates the problem of acquiring a set of measurements comparatively much smaller than dictated by Shannon‘s theorem, from which the signal can be reconstructed exactly or almost exactly. The main objective of CS is not to perform compression; rather, CS aims at avoiding altogether the acquisition of a very large number of samples, thereby allowing to design sensors that are more effective at acquiring the signal of interest. Interestingly, CS can be carried out using surprisingly simple techniques. A ”measurement” is obtained as the scalar product of the signal with a pseudorandom sequence. The reconstruction of the signal requires to find the sparsest signal (or transform thereof) that matches the available measurements, which can be performed using, amongst others, linear programming techniques. The goal of this work is to provide a proof-of-concept of CS technology for satellite imaging, allowing to design simpler and cheaper sensors that provide a given resolution, or sensors with resolution higher than the number of detectors. This paper provides a *proof-of-concept*, an extremely important stage of the research activity: the development of an actual CS-based sensor is indeed a costly effort, which has to be justified by a thorough preliminary qualitative and quantitative analysis of the potential benefits and drawbacks.

Hyperspectral images

Satellite imaging is a highly effective tool in a variety of scientific and engineering contexts because of the information it provides about the nature of the materials being imaged. While traditional digital imaging techniques produce images with scalar values associated with each pixel location, in multi- and hyperspectral images these values are replaced with a vector containing the spectral information associated to that spatial location. The resulting image is therefore three-dimensional (two spatial and one spectral dimensions), and spectral resolution is very important for several applications, including classification, anomaly detection, and spectral unmixing. Despite the huge potential, however, many modern satellite imagers face a limiting trade off between spatial and spectral resolution. In fact, the total number of samples that can be acquired is constrained by the size of the detector array. This limits the usefulness and costeffectiveness of spectral imaging for many applications. This work intends to overcome this limitation by investigating a new imaging architecture based on CS. It will address the design of CS strategies for an acquisition system that does not detect single pixels of the scene, but rather a small number of ”measurements”. Reconstruction of the image is going to be performed at the ground station, and all subsequent processing steps (radiometric and geometric calibration, orthorectification, and applications) would be performed on the reconstructed image. It is foreseen that a CS-based system will provide the following advantages over a conventional system.

- The resulting imaging system will require a smaller detector area for the same spatial or spectral resolution, and for the same image quality. This will yield a cheaper system, enabling low-cost missions to take advantage of spectral imaging technology.

- Alternatively, the resulting system will provide significantly increased spatial or spectral resolution for the same detector area. this will allow to extract more accurate spectral information from the image.

- The resulting imaging system will be mechanically simple and robust, as it does not require to use moving parts (e.g., mirrors) as some conventional systems do, such as whiskbroom scanners.

- For equal resolution, the decrease in the number of acquired measurements with respect to a conventional system will yield lower sensor data rates. This will facilitate data handling by the onboard processing unit, allowing to employ cheaper components and to save mass.

- Moreover, the reduced data rate will significantly facilitate data transmission to the ground segment, allowing to employ less bandwidth.

This work intends to advance the state-of-the-art in the field of satellite imaging through application of the CS paradigm. In the long term, it is expected that CS will lead to a new generation of imaging systems, capable to achieve extremely high resolutions with reasonable cost, and to achieve state-of-the-art resolution with reduced cost. Therefore, ambitious as well as low-cost missions can benefit from the potential advantages yielded by CS.

All the results and description in this paper have been derived considering two set of images, from two of the most famous and used satellite imaging sensors: AVIRIS (224 bands, see [11]), and AIRS (1501 bands, see [1]).

Compressive Sampling

CS, in summary, consists in retrieving a signal f with dimension N by only using $M << N$ sampled measurements, without any information loss (see [4]). This is possible if f has a sparse or compressible representation θ in some basis Ψ (i.e. $f = \Psi\theta$). The theory is indeed based on two main principles (see [4]): *sparsity*, and *incoherence*. Sparsity can be defined as the number of non-zero samples (or close to zero) and is a property of the signal of interest. A crucial fact is that the best sparsity value for a signal could be in a different domain respect to the original signal domain. Incoherence pertains to the sensing modality: if the signal is sparse in a certain basis Ψ, it has to be spread in a domain of acquisition Φ. Random matrices are largely incoherent with any fixed basis Ψ, and this is very important for the CS theory in practice.

Recovery of the original signal is made by an l_1-norm minimization process (see [4] and [3]). It is possible,without noise, to have perfect reconstruction if the matrices Φ and Ψ are perfectly incoherent with a number of sampling measurements $M = S * logN$(see [4]).

In this paper we exploit the potential of CS theory for Hyperspectral images by using spectral (1D) and spatial (2D) correlation, one by one and together (3D sampling). For the three dimensional case, the great complexity of the reconstruction process is reduced thanks to the Kronecker product (*KCS*, see [10]).

We provide here a more deep mathematical description of Compressive Sampling:

Notation and definitions

We denote (column-) vectors and matrices by lowercase and uppercase boldface characters, respectively. The n- element of a vector $\mathbf{x}$ is $(\mathbf{x})_n$. The (m, n)-th element of a matrix $\mathbf{A}$ is $(\mathbf{A})_{mn}$. The transpose of a matrix $\mathbf{A}$ is $\mathbf{A}^\mathsf{T}$. The notation $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product of $\mathbf{A}$ times $\mathbf{B}$. The notation $\mathrm{vec}\{\mathbf{A}\}$ denotes the column vector obtained by stacking the columns of $\mathbf{A}$ on top of each other from left to right. The notation $\|\mathbf{x}\|_{\ell_0}$ denotes the number of nonzero elements of vector $\mathbf{x}$. The notation $\|\mathbf{x}\|_{\ell_1}$ denotes the ℓ_1-norm of the vector $\mathbf{x}$ and is defined as $\|\mathbf{x}\|_{\ell_1} \triangleq \sum_i |(\mathbf{x})_i|$.

The notation $\|\mathbf{x}\|_{\ell_2}$ denotes the Euclidean norm of the vector $\mathbf{x}$ and is defined as $\|\mathbf{x}\|_{\ell_2} \triangleq \sqrt{\sum_i |(\mathbf{x})_i|^2}$. The notation $a \sim \mathcal{N}(\mu, \sigma^2)$ means that the random variable a is Gaussian distributed, its mean value is μ, and its variance is σ^2

General problem

The general problem we wish to solve can be stated as follows: we want to recover an unknown vector $\mathbf{f} \in \mathbb{R}^{N\times 1}$ from a smaller vector $\mathbf{y} \in \mathbb{R}^{M\times 1}$, $M < N$, of linear measurements

$$C\mathbf{y} = \mathbf{\Phi}\mathbf{f} \; , \tag{1}$$

where $\mathbf{\Phi} \in \mathbb{R}^{M\times N}$ is the *sensing matrix*.

We assume that $\mathbf{f}$ is a compressible signal that can be represented by a quasi–sparse vector $\mathbf{x} \in \mathbb{R}^{N\times 1}$ of coefficients in a convenient orthonormal basis $\mathbf{\Psi} \in \mathbb{R}^{N\times N}$ such that

$$C\mathbf{x} = \mathbf{\Psi}^{\mathsf{T}}\mathbf{f} \; . \tag{2}$$

In the case $\mathbf{f}$ is a compressible signal, its representation $\mathbf{x}$ in the orthonormal basis $\mathbf{\Psi}$ is very concise, i.e. it shows relatively few $K < N$ significant elements capturing almost all the energy of $\mathbf{f}$.

Recovery

In the standard CS framework, introduced in [5, 8, 9], the recovery of $\mathbf{x}$ (and consequently of $\mathbf{f}$), can be obtained solving the following linear programming problem

$$C't'C\mathbf{x}_{\mathsf{CS}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} \quad subjectto \quad \mathbf{U}\mathbf{x} = \mathbf{y} \; , \tag{3}$$

where $\mathbf{U} = \mathbf{\Phi}\mathbf{\Psi}$. Linear programming is not the only possible recovery strategy, but it does tend to make $\mathbf{x}_{\mathsf{CS}}$ sparse.

When the measures are noisy, like in the case of quantized data which are subject to quantization noise, the ℓ_1 minimization with relaxed constraints is used for reconstruction:

$$C't'C\mathbf{x}_{\mathsf{CS}} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|_{\ell_1} \quad subjectto \quad \|\mathbf{U}\mathbf{x} - \mathbf{y}\|_{\ell_2} < \varepsilon \; , \tag{4}$$

where ε bounds the amount of noise in the data.

The recovery of (3) or (4) is possible if the sensing matrix $\mathbf{\Phi}$ obeys the *Restricted Isometry Property* (RIP) of order K [7], which means that there exist constants $a, b \in R$, with $0 < a < b \leq \infty$, such that the following inequality

$$Ca \|\mathbf{x}\|^2_{\ell_2} \leq \|\mathbf{\Phi}\mathbf{x}\|^2_{\ell_2} \leq b \|\mathbf{x}\|^2_{\ell_2} \tag{5}$$

holds for all $\mathbf{x}$ such that $\|x\|_{\ell_0} \leq K$.

It has been shown in [2] that extracting the elements of $\mathbf{\Phi}$ at random from a normal distribution with

$$(\mathbf{\Phi})_{mn} \sim \mathcal{N}\left(0, \frac{1}{M}\right) , \tag{6}$$

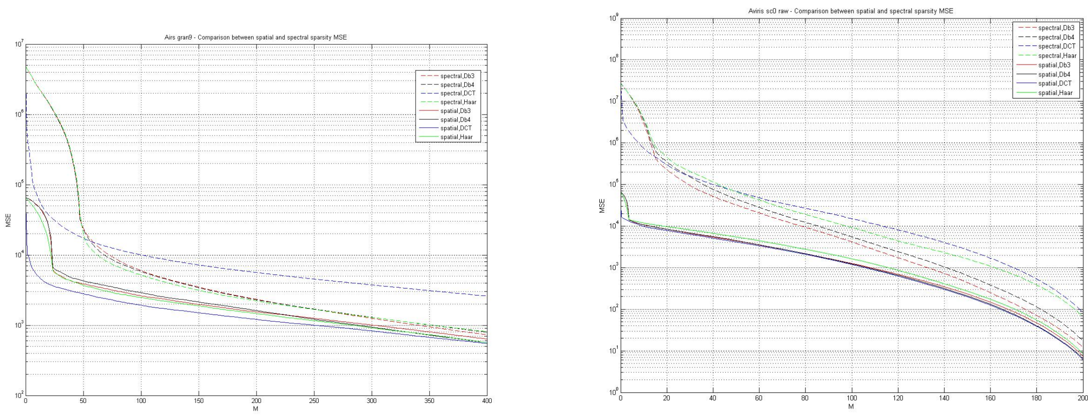
or from a Bernoulli distribution such that

$$\Pr\left((\mathbf{\Phi})_{mn} = \pm\sqrt{\frac{1}{M}}\right) = \frac{1}{2}, \tag{7}$$

cause the matrix $\mathbf{\Phi}$ to respect the RIP with overwhelming probability, provided that $M = O(\mu^2 K \log(N/K))$, where $\mu = \max_{i,j}\{|\mathbf{U}_{ij}|\}$. A strictly sparse signal is recovered perfectly with overwhelming probability [7], i.e. $\mathbf{\Psi}\mathbf{x}_{\mathsf{CS}} = \mathbf{f}$, while the recovery of a quasi–sparse signal is as good as if one knew ahead of time the locations of the K largest coefficients of $\mathbf{x}$ [6].

PAPER OVERVIEW

After a brief description of the state-of-the-art, we describe our testbed, giving then results.



(a) (b)

Figure 1: Comparison of the spatial and spectral sparsity curves for the airs and the aviris case.

PREVIOUS WORK

Even if Compressive sampling theory is relatively recent, there is already a great variety of literature and applications.

TESTBED DESCRIPTION

We made tests on Airs and Aviris Hyperspectral images(see [11] and [1]): firstly we have tried to give a value to the sparsity of the images in different domains (wavelets db1, db3, db4, and dct transform). Then, having sparsity values, we applied CS theory: we acquired the signals varying the number of measurements M, using random matrices as acquisition domains, following [4].

The last step is the reconstruction process: we used the minimization algorithm proposed in [3]. We analyzed the two signals both in the spectral dimension (with a 32×32 pixels window) and in the spatial dimension, to see where the correlation is higher. The three dimensional case is performed in a Kronecker product in dct basis with KCS schema (see [10]). Spatial and spectral results are averaged between pixels for the spectral case, and between bands for the spatial case. Obviously 3D case is not averaged.

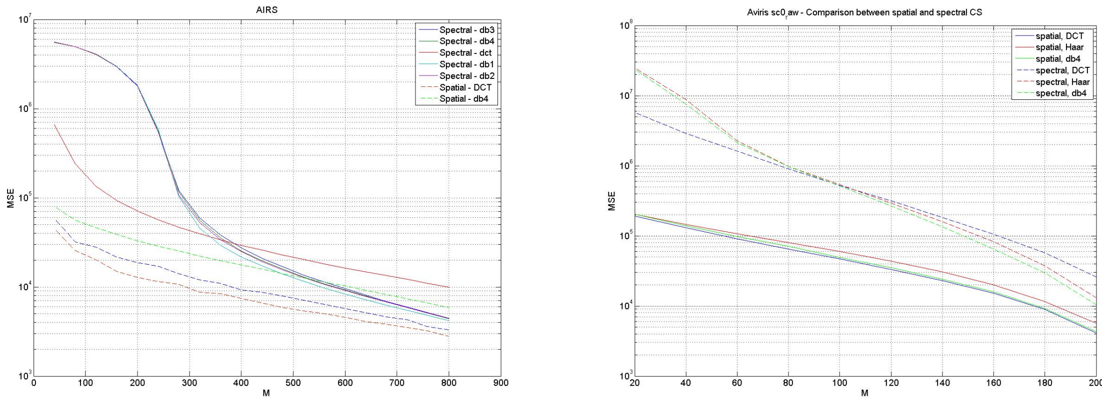
Sparsity Evaluation

To calculate the sparsity for different domains, we transformed the signals (i.e. the hyperspectral images) using wavelets db1, db3, db4, and dct. We sorted then the samples in ascending order, throwing away an increasing number of samples starting with the less significant ones. Finally, after each step of throwing samples, we applied the inverse transforms to calculate MSE and SNR values. Therefore from this process we have the curve in 1. In the three dimensional case, we applied only the dct transform, processing the entire data with a basis formed from the Kronecker product of a 2D dct basis in space and a 1D dct basis along the spectrum.

For the AVIRIS and AIRS images, we used two voxels of dimension $N = 32 \times 32 \times 224$ and $N = 32 \times 32 \times 1501$, respectively.

CS theory application

To apply the CS theory as descripted above, the only step is to multiply the measurements' $\mathbf{f}$ by the Φ random matrix (see equation 1).



(a) (b)

Figure 2: Comparison of the spatial and spectral result curves for the airs and the aviris case.

Recovery

We compare first the performance of spatial CS to spectral CS and then the global performance of CS to KCS using the DCT basis 2D to sparsify individual band images and the DCT basis 1D to sparsify all pixels along the spectral dimension. In our experiments we obtain 1D and 2D CS measurements $\mathbf{f}$ from the sparsity figures using a matrix Φ along the spectral and spatial dimension on an image of size $32 \times 32 \times 1501$. The Φ matrix has to satisfy the Restricted Isometry Property (see equation 6). We also obtain global CS measurements that depend on all the voxels of the hyperspectral images; such measurements result in a fully dense measurement matrix and therefore are difficult to obtain in real world applications. For this, in the 3D recovery with KCS,the data used in the 1D and 2D recovery with CS was *flattened* to $16 \times 16 \times 1024$ voxels to reduce the amount of computation required. Figure 2 shows the comparison between the spectral and spatial CS recovery error MSE. In the 1D-2D case, CS recovery operates along the spectral spatial dimension with the measurement matrix $\mathbf{f}$ using different transforms (the three wavelets basis and one DCT) to sparsify the signal. In the 3D case, KCS operates on each spectral band with the same measurement matrix $\mathbf{f}$ to acquire measurements and employs the Kronecker product measurement matrix $I \times \mathbf{f}$ to perform joint recovery. We used a Kronecker sparsifying basis as a Kronecker products of a 1D Fourier basis in the spectral dimension and a 2D Fourier basis in the spatial dimensions. Figure **??** shows results using these two Kronecker products sparsifying basis together with KCS measurements as Kronecker products matrix. We see again that KCS recovery outperforms spectral and spatial CS recovery operating on the same measurements.

RESULTS

We have firstly taken into consideration the 1D and 2D cases, then also the three dimensional case was treated.

Figure 2 shows comparison between spatial and spectral sparsity MSE versus S values. We see that the spatial trasforms provide the sparsest representation of signals. In both the 1D and 2D cases, the representation of the signal in the the Fourier basis is more sparse than the representation of signals in the three wavelet Daubechies. From the Figure 2 we see that the sparsity values for the spectral image AIRS is in the range 20:25, i.e we used only 2% of Fourier coefficients to represent the signal with an MSE error equals to 5.8×10^7 e 4.66×10^7.

Our results are coherent with CS theory, obtaining similar curves with a value of $M > SlogN$ (see Figure ...).

In the AIRS case, we have seen that in one dimension we have 2% of significant coefficients, with an MSE error of about 2×10^4

CONCLUSION

It seems that better performances are given by the spatial correlation respect to the spectral one. In the AVIRIS case 224 bands are not enough to evidence benefits using the CS procedure in the spectral case. Instead in the AIRS case we reach MSE values that are sufficiently good for many applications with around 400 measurements over 1501 (spectral case). Even better results are reached in the spatial case. The 3D correlation provides the best results, even if tests was made only in the dct case.

FUTURE WORK

More tests with 3D CS.

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