The Method of Types: a useful technical tool for forensic analysis

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Outline

- **Theoretical background**
  - Description of the Method of Types;
  - Universal Source Coding.
- **An application to Multimedia Forensics**
  - Adversary-aware source identification: the known source case.

Theoretical background

Description of the Method of Types

Introduction

- Csiszár, Körner (1981).
- **Powerful tool in Information Theory (IT)**:
    - all the most important results of IT can be proved by using the M.of T. : *Shannon theory*, *AEP (large deviation theory)*, *channel capacity*,....;
    - the **Universal Source Coding** wholly relies on the M. of T.
- Based on elements of *combinatorial calculus*.

The concept of 'type'

$X \rightarrow$ Source of symbols, DMS ($\mathcal{X} \rightarrow$ alphabet);
a_i, $i = 1, 2...|\mathcal{X}| \rightarrow$ symbols;
$X^n \rightarrow$ random sequence of length n;
$x^n \rightarrow$ realization of X^n, n-length vector drawn from the source;

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Definition (Type)

The type of a sequence x^n is the empirical probability distribution (dpe), i.e. the probability distribution for the source X we are able to estimate from the available sequence,

$$P_{x^n} : \mathcal{X} \rightarrow [0, 1] \qquad P_{x^n}(a_i) = \frac{N(a_i/x^n)}{n} \qquad \forall a_i,\ i = 1, 2, ..., |\mathcal{X}|.$$

$P_{x^n} \rightarrow |\mathcal{X}|$-length vector

Some notation and basic concepts

- $\mathcal{P}_n$: the set of all types computed on n-length sequences: $\mathcal{P}_n = \{P_{x^n}\}$;
- $T(P_{x^n})$: the set of n-length sequences having type P_{x^n}: $\forall P \in \mathcal{P}_n,\ T(P) = \{x^n : P_{x^n} = P\}$; $T() \rightarrow$ type class;

$\Rightarrow$ P_{x^n}, $\mathcal{P}_n$ and $T(P_{x^n})$ are the 'actors' of the Method of Types.

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$\Rightarrow$ P_{x^n}, $\mathcal{P}_n$ and $T(P_{x^n})$ are the 'actors' of the Method of Types.

Quantity of IT involved: the K-L distance and Empirical entropy.

Remind

- Kullback-Leibler distance or divergence between two distributions (e.g. P and Q) on the same alphabet:

$$\mathcal{D}(P||Q) = \sum_{a \in \mathcal{X}} P(a) \log \frac{P(a)}{Q(a)};$$

- Empirical entropy: $H(P_{x^n}) = -\sum_{a \in \mathcal{X}} P_{x^n}(a) \log P_{x^n}(a)$.

The Method of Types

The Method of Types : provides useful bounds on the probability of a type class $Pr\{T(P)\} = Pr\{x^n \in T(P)\}$, for any $P \in \mathcal{P}_n$, and states its behavior for large n (*strong version of the LLN*).

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Observation (Cardinality of $\mathcal{P}_n$)

$|\mathcal{P}_n| < (n+1)^{|\mathcal{X}|}$

Outline: any 'individual' symbol a_i has $(n+1)$ different occurrences possible.

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Observation (Cardinality of $T(P)$)

The number of n-length sequences having type P has the following bounds:

$$\frac{2^{nH(P)}}{(n+1)^{|\mathcal{X}|}} \leq |T(P)| \leq n2^{nH(P)}. \quad (1)$$

$X \sim Q(x)$

Given a sequence x^n drawn from the source:

- $Pr\{x^n\}$?

OSS: $Pr\{x^n\}$ is the same for all the sequences x^n belonging to the same type class.

Theorem (Probability of a sequence)

The probability of a sequence x^n having type P_{x^n} is

$$Pr\{x^n\} = 2^{-n[H(P_{x^n}) + \mathcal{D}(P_{x^n}||Q)]}. \quad (2)$$

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Proof (1/2).

$$Pr\{x^n\} = \prod_{i=1}^{n} Q(x_i) = \prod_{a \in \mathcal{X}} Q(a)^{N(a/x^n)}$$

(2/2).

$$
\begin{aligned}
&= \prod_{a\in\mathcal{X}} Q(a)^{\frac{N(a/x^n)}{n}\cdot n} \\
&= \prod_{a\in\mathcal{X}} Q(a)^{P_{x^n}(a)\cdot n} \\
&= \prod_{a\in\mathcal{X}} 2^{nP_{x^n}(a)\log Q(a)} \\
&= \prod_{a\in\mathcal{X}} 2^{n[P_{x^n}(a)\log Q(a) - P_{x^n}(a)\log P_{x^n}(a) + P_{x^n}(a)\log P_{x^n}(a)]} \\
&= 2^{n\left(\sum_a [P_{x^n}(a)\log Q(a) - P_{x^n}(a)\log P_{x^n}(a) + P_{x^n}(a)\log P_{x^n}(a)]\right)} \\
&= 2^{-n[H(P_{x^n}) + \mathcal{D}(P_{x^n}||Q)]}. \qquad (3)
\end{aligned}
$$

□

Corollary

If $Q = P_{x^n}$, then

$$Pr\{x^n\} = 2^{-nH(P_{x^n})}. \qquad (4)$$

The corollary allows founding a stricter upper bound for $|T(P)|$, by simply noting that

$$Pr\{T(P_{x^n})\}_{Q=P_{x^n}} = |T(P_{x^n})| \cdot 2^{-nH(P_{x^n})} \leq 1$$

$$\rightarrow |T(P_{x^n})| \leq 2^{nH(P_{x^n})}.$$

Hence, $\forall P \in \mathcal{P}_n \qquad \frac{2^{nH(P)}}{(n+1)^{|\mathcal{X}|}} \leq |T(P_{x^n})| \leq 2^{nH(P)}.$

Given a type $P \in \mathcal{P}_n$:

- $Pr\{T(P)\}_Q$?

Theorem (Probability of a type class)

The probability of the type class $T(P)$ is bounded as follows:

$$\frac{2^{-n\mathcal{D}(P||Q)}}{(n+1)^{|\mathcal{X}|}} \leq Pr\{T(P)\}_Q \leq 2^{-n\mathcal{D}(P||Q)}. \quad (5)$$

Proof (1/2).

$$Pr\{T(P)\}_Q = |T(P)| \cdot Pr\{x^n\} = |T(P)| \cdot 2^{-n[H(P)+\mathcal{D}(P||Q)]}. \quad (6)$$

(2/2).

We use the known bounds on $|T(P)|$:

1. $Pr\{T(P)\}_Q \leq 2^{nH(P)} \cdot 2^{n[H(P_{x^n}) + \mathcal{D}(P_{x^n}||Q)]} = 2^{-n\mathcal{D}(P||Q)}$;
2. $Pr\{T(P)\}_Q \geq \frac{2^{nH(P)}}{(n+1)^{|\mathcal{X}|}} \cdot 2^{n[H(P_{x^n}) + \mathcal{D}(P_{x^n}||Q)]} = \frac{2^{-n\mathcal{D}(P||Q)}}{(n+1)^{|\mathcal{X}|}}$.

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According to the theorem, at the first order on the exponent we have

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- *For large n ?*

⇒ The *Law of Large Numbers (LLN)* comes out!

Hence, the unitary sum constraint yields: $Pr\{T(Q)\}_Q \to 1$.

[1] This is always possible if n is sufficiently large.

⇒ The *Law of Large Numbers (LLN)* comes out!

In order to see this interesting result, let us consider that

- If $Q \in \mathcal{P}_n$ [1] we can write

$$Pr\{T(Q)\}_Q \leq 1;$$

- As to the others $P \in \mathcal{P}_n$:

$$\begin{aligned} Pr\left\{\begin{matrix}\text{tutte le altre}\\ \text{type classes}\end{matrix}\right\} &\leq \sum_{P \in \mathcal{P}_n, P \neq Q} 2^{-n\mathcal{D}(P||Q)} \\ &\leq (n+1)^{|\mathcal{X}|} \max_{P \in \mathcal{P}_n, P \neq Q} 2^{-n\mathcal{D}(P||Q)} \\ &\leq (n+1)^{|\mathcal{X}|} 2^{-n \min_{P \in \mathcal{P}_n, Q \neq P} \mathcal{D}(P||Q)} \to 0. \end{aligned}$$

Hence, the unitary sum constraint yields: $Pr\{T(Q)\}_Q \to 1$.

[1] This is always possible if n is sufficiently large.

To sum up, from the previous theorem follows that

"As n tends to infinity, the probability of the right type class, i.e. $Pr\{T(Q)\}_Q$, tends to 1 , while the probability of any other type class or wrong type class, i.e. $Pr\{T(P)\}_Q$ (with $P \neq Q$), tends to 0"; that is, as $n \to \infty$

- $Pr\{T(Q)\}_Q \to 1$;
- $Pr\{T(P)\}_Q \to 0$.

OSS: the *decreasing velocity* of each probability ($Pr\{T(P)\}_Q$, $P \neq Q$) is regulated by $\mathcal{D}(P||Q)$.

This result can be interpreted as follows:

- The number of the sequences, i.e. the "right" and "wrong" ones[2], grows much with n; that is

$$|T(P)| \simeq 2^{nH(P)};$$

  Some "wrong-type" sequences could be in number more than the "right-type" ones;
- the probability of a sequence decreases very rapidly as n increase, according to $Pr\{x^n \in T(P)\}_Q = 2^{-n(H(P)+\mathcal{D}(P||Q))}$.

Thus, for a given type class, "**the only way through which the increasing of the number of sequences could balance the reduction of the probability of any sequence is $\mathcal{D}(P||Q) = 0$, which can only be achieved if $P = Q$**".

[2] "right sequence" $= x^n \in \{T(Q)\}_Q$, "wrong sequence" $= x^n \in \{T(P)\}_Q$ where $P \neq Q$.

Universal Source Coding

Universal Source Coder (Weak)

The weak Universal Source Coder is a coder which, employing a bit rate R, succeeds in correctly coding any source $X \sim Q(X)$ having $H(X) \leq R$.

- Why Weak?

If the source has $H(X) < R$ the universal coder does not reach the entropy as code rate (Shannon Coding) $\rightarrow$ *it's possible to do better!*

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Universal Source Coder (Strong)

The strong Universal Source Coder is a coder which, for any source $X \sim Q(X)$, succeeds in generating a code having rate $R = H(X)$.

- Why Strong?

$H(X)$ is the Shannon limit for the rate $\rightarrow$ *it's NOT possible to do better!*

- Such a coder (weak and strong) really exists?

Yes, the Method of Types allows to prove the existence.

Theorem (Existence of the Weak U.S.C.)

For any discrete memoryless source X a Universal Source Coding exists.

Outline of the Proof (1/2).

We are interested in *coding* the sources X with $H(X) \leq R$. Let us show it is possible by using rate R.

$$\text{Fix } n,\ \mathcal{P}_n \rightarrow \begin{cases} P : H(P) \leq R & (a) \\ P : H(P) > R & (b). \end{cases}$$

We consider only types P in (a) [3].

[3] It is reasonable according to the Method of Types.

(2/2).

- How many bit are necessary?
  - How many sequences there are in a **type-class**?

$$|T(P)| \leq 2^{nH(P)} \leq 2^{nR};$$

  - How many **type**?

$$N^{\circ}\{P \in (a)\} \leq |\mathcal{P}_n| < (n+1)^{|\mathcal{X}|}.$$

Number of sequences which must be indexed: $< (n+1)^{|\mathcal{X}|} 2^{nR}$.

Average number of *bit per symbol* required:

$< \frac{\log[(n+1)^{|\mathcal{X}|} 2^{nR}]}{n} = |\mathcal{X}| \frac{\log(n+1)}{n} + \frac{nR}{n} \longrightarrow R$ bit/symbol.

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- Practical coders: *LZ77*, *LZ78*, *LZW*.

Source Coding vs Universal Source Coding

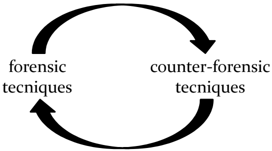
- Are both asymptotic codings!
- The differences lies on the *velocities*.

  If the source is known (Source Coding) the entropy value can be reached *by a lower n in practice*.

An application to Multimedia Forensics

Definition of the problem

Limits of forensic analysis in presence of an adversary: "*Any attempt to improve the forensic analysis will be accompanied by a dual effort to device more powerful counter-forensic techniques that leave less and less evidence into the forged documents*"
$\rightarrow$ **virtuous loop**



Compelling goal: to investigate the ultimate limits of forensics and counter forensics analysis.

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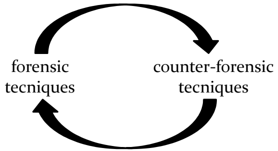

Compelling goal: to investigate the ultimate limits of forensics and counter forensics analysis.

- Our contribution: to provide a theoretical framework to the **source identification problem** in presence of an adversary.

Adversary-aware source identification: the known source case

The Source Identification Problem

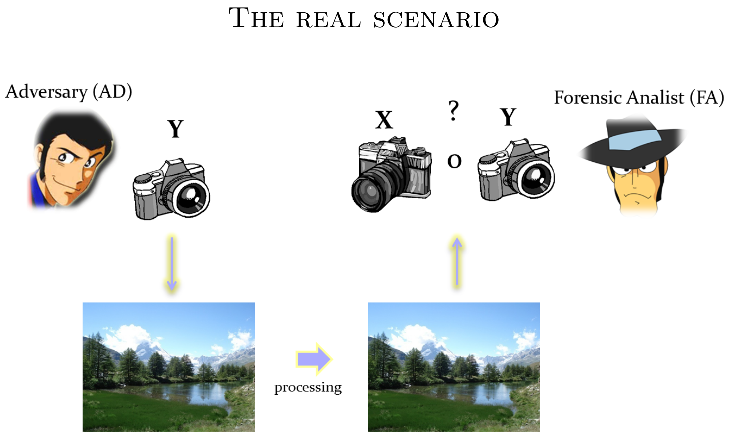

Figure : the image the AD want to modify might have *critical relevance* in many fields (e.g. judicial, medical,....).

Problem schematization

Sources: $X \sim P_X$, $Y \sim P_Y$.

The FA's aim: to distinguish sequences generated by X from those generated by Y.

$$x^n = x_1, x_2, \ldots, x_n \quad \rightarrow \ \in X \ \text{ or } \in Y?$$

The AD's aim: to trasform a sequence drawn from Y, e.g. y^n, into a new sequence z^n, as close as possible to y^n [a], in such a way that the FA believes that z^n has been generated by X.

$$y^n = y_1, y_2, \ldots, y_n \quad \rightarrow \quad z^n = z_1, z_2, \ldots, z_n.$$

[a] in real contexts the AD will want to preserve perceptual similarities between the images.

Theoretical foundations

- **Game Theory** $\rightarrow$ the source identification problem is formalized as a game (*zero-sum game*).

Players: the Forensic analyst (FA) and the Adversary (AD).

Game analysis: the main theoretical tools are

- **Hypothesis test** : used to formalize the classification problem faced by the FA:

  $$Hp\ 0 = \text{"}x^n \text{ belongs to } X\text{"};$$
  $$Hp\ 1 = \text{"}x^n \text{ belongs to } Y\text{"};$$

- **Information theory**: is the branch to which the main quantities involved in our analysis belong.

The Source Identification Game (known source case)

→ *The FA and the AD know the source X.*
The source Y is known to the AD and not necessarily to the FA.

SI_{ks} game $\doteq (\mathcal{S}_{FA}, \mathcal{S}_{AD}, u)$
$\mathcal{S}_{FA}, \mathcal{S}_{AD} \rightarrow$ *sets of strategies,* $\quad u \rightarrow$ *payoff.*

$$\mathcal{S}_{FA} = \{\Lambda_0 : P_X(x^n \notin \Lambda_0) \leq P^*_{fp}\},$$
$$\mathcal{S}_{AD} = \{f(y^n) : d(y^n, f(y^n)) \leq nD\},$$
$$u = -P_{fn} = -P_Y(f(y^n) \in \Lambda_0) = -\sum_{y^n : f(y^n) \in \Lambda_0} P_Y(y^n).$$

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⇒ Limitations to the model for mathematical tractability:

hp) Asymptotic version of the game and limited resources for the FA: SI^{lr}_{ks}.

SI_{ks}^{lr} game: resolution procedure

1. Optimum strategies: $\mathcal{S}_{FA}^{*}$, $\mathcal{S}_{AD}^{*}$;
2. The profile $(\mathcal{S}_{FA}^{*}, \mathcal{S}_{AD}^{*})$ is a *Nash equilibrium*;
3. Payoff at the equilibrium: u^{*} $(= u(\mathcal{S}_{FA}^{*}, \mathcal{S}_{AD}^{*}))$.

$\rightarrow$ Step 1.
The determination of the optimum strategies passes through the search for the optimum acceptance region Λ_0 *and* f *function*:
$(\mathcal{S}_{FA}^{*}, \mathcal{S}_{AD}^{*}) \leftrightarrow (\Lambda_0^{*}, f^{*})$.

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The determination of the optimum strategies passes through the search for the optimum acceptance region Λ_0 *and* f *function*: $(\mathcal{S}_{FA}^*, \mathcal{S}_{AD}^*) \leftrightarrow (\Lambda_0^*, f^*)$.

• Consequence of the limited resources assumption (**lr**):

the acceptance region Λ_0 is a union of type classes!!

The set of strategies for the FA becomes:
$\mathcal{S}_{FA} = \{\Lambda_0 \in 2^{\mathcal{P}_n} : P_{fp} \leq 2^{-\lambda n}\}$.

Optimum strategy for the FA

- Λ_0^*?

The **Method of Types** allows to prove the following lemma:

Lemma (Optimum acceptance region)

Let Λ_1^* $(= \Lambda_0^{*,c})$ *be:*

$$\Lambda_1^* = \{P \in \mathcal{P}_n : \mathcal{D}(P||P_X) \geq \lambda - |\mathcal{X}|\frac{log(n+1)}{n}\}. \tag{7}$$

Then, we have

- $P_{fp} \leq 2^{-n(\lambda-\delta_n)}$, *with* $\delta_n \to 0$ *for* $n \to \infty$,
- *for every* $\Lambda_0 \in \mathcal{S}_{FA}$ *we have* $\Lambda_1 \subseteq \Lambda_1^*$.

Proof (1/2).

- *Part 1*

Λ_0^* and Λ_1^* are unions of type classes

$$P_{fp}(\Lambda_0^*) = P_X(x^n \in \Lambda_1^*) = \sum_{P \in \Lambda_1^*} P_X(T(P)). \quad (8)$$

By using the bound on the total number of types $|\mathcal{P}_n|$ and on the probability of a type class $Pr\{T(P)\}$, we have

$$\begin{aligned} P_{fp}(\Lambda_0^*) &\leq (n+1)^{|\mathcal{X}|} \max_{P \in \Lambda_1^*} P_X(T(P)) \\ &\leq (n+1)^{|\mathcal{X}|} 2^{-n \min_{P \in \Lambda_1^*} \mathcal{D}(P||P_X)} \\ &\leq (n+1)^{|\mathcal{X}|} 2^{-n\left(\lambda - |\mathcal{X}| \frac{\log(n+1)}{n}\right)} \\ &= 2^{-n\left(\lambda - 2|\mathcal{X}| \frac{\log(n+1)}{n}\right)}, \end{aligned} \quad (9)$$

(2/2).

proving the first part of the lemma with $\delta_n = 2|\mathcal{X}|\frac{\log(n+1)}{n}$.

- *Part 2*

Take an arbitrarily region $\Lambda_0 \in \mathcal{S}_{FA}$ and let P be a type in Λ_1:

$$\begin{aligned} 2^{-\lambda n} &\geq P_X(\Lambda_1) \\ &\geq P_X(T(P)) \\ &\geq \frac{1}{(n+1)^{|\mathcal{X}|}} 2^{-n\mathcal{D}(P||P_X)}. \end{aligned} \tag{10}$$

Hence, by taking the log of both sides:

$$\mathcal{D}(P||P_X) \geq \lambda - |\mathcal{X}|\frac{log(n+1)}{n}, \tag{11}$$

proving that $P \in \Lambda_1^*$. □

Interesting consequence of the *Lemma*:

The FA optimum strategy does not depend on:

- the strategy chosen by the AD;
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The optimum strategy is universally optimal across all the probability density function.

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- *To sum up* :

The Method of Types has given a valuable contribution to our analysis by providing the **optimum strategy for the FA** (*first step of the Game Analysis*).

Future steps

The Method of Types turns out to be a useful tool even for

- determining the *value* and the *behavior* of the payoff at the equilibrium, $u(\Lambda_0^*, f^*)$ (*third step of the Game Analysis*);
- retracing the same steps and solving the **Source Identification Game with Training Data**.

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