AN INTRODUCTION TO GAME THEORY

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Contents

- An overview:
  - What is Game Theory?
  - Classification
- Non-cooperative games
  - Strategic games (with perfect information)
    - Definition
    - Examples of games in strategic form
    - Solution concepts:
      - Dominant Strategy equilibrium
      - Nash equilibrium
      - Refinements of the Nash equilibrium
  - Strategic games (with imperfect information) : Bayesian games

AN OVERVIEW

What is Game Theory?

- **Goals**

  It aims to help us understand situations in which *decision-makers* (*players*) interact: **Interactive Decision Theory**

- **Origins**

  'Theory of Games and Economic Behavior ' by von Neumann and Morgensten (1944)

- **Application Areas**

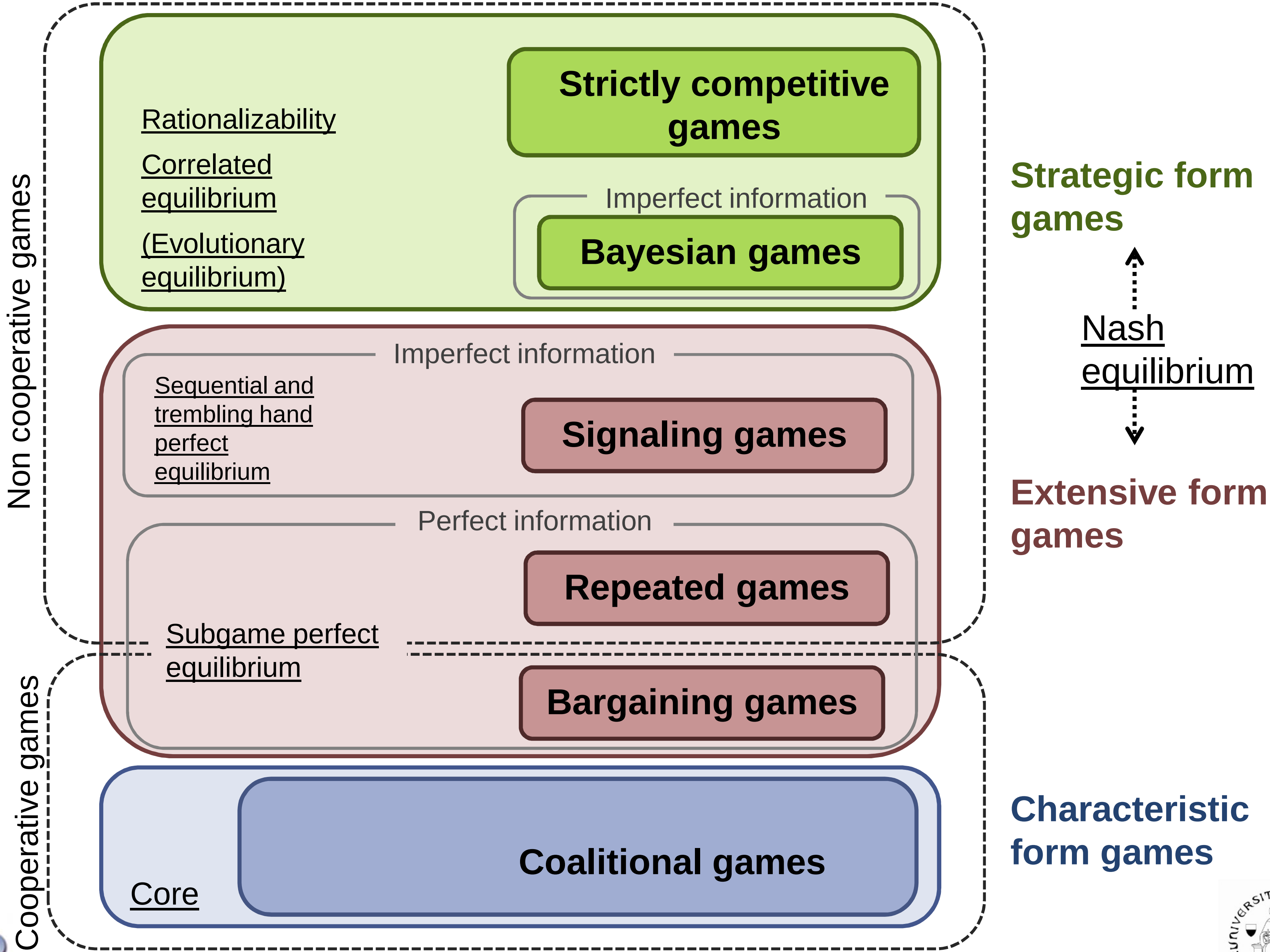
  Economics, political science, psychology, computer science

❑**Assumption:** the players are ***rational*** (have a clear relation of preferences over the outcomes[1]) and ***intelligent*** (are able to act in a rational way)

1 Axioms of "rationality" (**Von Neumann–Morgenstern utility theorem, 1947**)

Classification

- Non-Cooperative and Cooperative Game
  - *Non Cooperative:* binding agreements are not allowed
  - *Cooperative*: binding agreement are allowed
- Games with Perfect and Imperfect Information
  - *Games with Perfect Information*: the players are fully informed about the possible moves of the others players
  - *Games with Imperfect Information*: the players have only partial information about the possible moves of the others players
- Games in Extensive, Strategic and Characteristic form
  - *Extensive form*: detailed description of the game (before 1944)
  - *Strategic form*: game in normal form; Von Neumann-Morgenstern (1944)
  - *Characteristic form*: for cooperative games only


Strictly competitive games
Rationalizability
Correlated equilibrium
(Evolutionary equilibrium)
Imperfect information
Bayesian games
Strategic form games
Nash equilibrium
Imperfect information
Sequential and trembling hand perfect equilibrium
Signaling games
Extensive form games
Perfect information
Repeated games
Subgame perfect equilibrium
Bargaining games
Non cooperative games
Cooperative games
Core
Coalitional games
Characteristic form games

NON-COOPERATIVE GAMES

Strategic Games

Definition of Strategic Game

«A model of interaction among *decision makers*. Each player chooses his 'plane of action' *once and for all* and the choices are made simultaneously.».

$S = \times_{j \in N} S_j \quad (\succsim_i)$

$A_i,\ i \in N \Leftrightarrow u_i : S$

- a finite set N (***players***)
- for each player $i \in N$
  - a nonempty set $S_i = \{s_i^1, s_i^2, ...\}$ (set of ***strategies*** available to i)
  - a preference relation ($\succsim_i$) on $S = \times_{j \in N} S_j$ (set of *outcomes* or *profiles*)
    - a profile s is a *N*-pla of strategies $s = (s_j^{k(j)})_{j \in N}$

a *preference relation* $\succsim_i$ *is a function* $u_i : S \to \mathbb{R}$ ***(payoff)***

$$s_1 \succsim_i s_2 \Leftrightarrow u_i(s_1) \geq u_i(s_2)$$

Games in strategic form: examples

Prisoner's Dilemma

I/II	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

Battle of sexes

him/her	$football$	$opera$
$football$	2,1	0,0
$opera$	0,0	1,2

Head and Tail

I/II	H	T
H	-1,1	1,-1
T	1,-1	-1,1

Pure Coordination

I/II	$football$	$rugby$
$football$	1,1	0,0
$rugby$	0,0	1,1

Non-cooperative strategic games:

- **one-shot** games
- ***repeated*** *games (the strategic model is appropriate only if there are no strategic ties among the repetitions)*

Some notation and definitions

- <u>Some notation</u>
  - If $s = (s_i)_{i \in N}$ is a *strategy profile,* then $s_{-i} = (s_1, ..., s_{i-1}, s_{i+1}, ..., s_N)$
  - $(s_i, s_{-i}) = s$

- <u>Definitions</u>
  - s_i is a **best response** to s_{-i} if

    $u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ *for every strategy* s_i' *available to i*

  - s_i *is a* **unique best response** to s_{-i} if

    $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ *for every* $s_i' \neq s_i$

STRATEGIC GAMES

Solution Concepts

Solution concepts

- In Game Theory (multiple agents or players) the a 'best strategy' for a player depends on others' choices.

  Solution concepts = 'subsets of outcomes (profiles) which are in some sense preferable'.

- Some solution concepts (*non-cooperative strategic games*):
  - **Pareto optimality**
  - **Dominant Strategy equilibrium**
  - **Nash equilibrium**
  - **Iterated elimination of Strictly Dominated Actions (Rationalizablility)**
  - **Mixed strategies Nash equilibrium**
  - **Correlated equilibrium**

  } *non deterministic player's strategies*

Pareto optimality

- The strategy profile s **pareto dominates** a strategy profile s' if
  - no agent gets a worse payoff with s than with js'
    i.e. $u_i(s) \geq u_i(s')$ for all *i*
  - at least one agent gets a better payoff with s than with s'
    i.e. $u_i(s) > u_i(s')$ for at least one *i*

- A strategy profile s is **Pareto optimal** or **strongly Pareto efficient** if there is no strategy s' that Pareto dominates s
  - every game has at least one Pareto optimal profile
  - there is always at least one Pareto optimal profile in which the strategies are pure

Example

The Prisoner's Dilemma

- (NC,NC) is Pareto optimal
  - no profile gives both players a higher payoff
- (NC,C) is Pareto optimal
  - no profile gives player I a higher payoff (or at least equal)
- (C,NC) is Pareto optimal
- (C,C) is Pareto dominated by (NC,NC)

I/II	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

Dominant strategy

Definition

Let $S_i = \{s_i^1, s_i^2, ...\}$ the set of all the strategies available to agent *i*

- The strategy s_i^k **strongly dominates** s_i^h for player *i* if

$$u_i(s_i^k, s_{-i}) > u_i(s_i^h, s_{-i}) \quad \forall s_{-i}$$

player i always does better with s_i^k *than* s_i^h

- The strategy s_i^k **weakly dominates** s_i^h

$$u_i(s_i^k, s_{-i}) \geq u_i(s_i^h, s_{-i}) \quad \forall s_{-i}$$

$$u_i(s_i^k, s_{-i}) > u_i(s_i^h, s_{-i}) \quad \text{for some } s_{-i}$$

player i never does worse with s_i^k *than* s_i^h *and there is at least one case in which he does better*

- s_i^k is a (**strongly,weakly**) **dominant strategy** if (strongly, weakly) dominates every $s_i^h \in S_i$

Dominant strategy equilibrium

- A **dominant strategy equilibrium** is a profile $S = (s_1, ..., s_N)$ such that s_i is *dominant* for the player *i*
- Each player *i* do best by using s_i rather than a different strategy. *regardless of what strategy the other playes use*.

Example (The Prisoner's Dilemma)

- there is one dominant strategy equilibrium: (C,C)
  - both player defect
  - it is not *Pareto optimal*

I/II	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

- *It is a* ***stronger concept than the Nash equilibrium***

Nash equilibrium

The most important solution concept for non-cooperative games

<u>Definition</u> *(pure strategy Nash equilibrium)*

A strategy profile $s^* = (s_1^*, ..., s_N^*)$ is a Nash equilibrium if for every player *i* if

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \qquad \text{for every } s_i \in S_i$$

i.e. for every player *i* s_i^* is the *best response* to s_{-i}^* / no player can yield an higher payoff by *unilaterally* changing his strategy.

- Interpretation: *steady state*
- **Dominant Strategy equilibrium** $\Rightarrow$ **Nash equilibrium**
  $\not\Leftarrow$

$$u_i(\bar{s}_i, \bar{S}^{-i}) \geq u_i(s_i, \bar{S}^{-1}) \quad \text{f}$$
$$u_i(s_i^*, S^{-i}) \geq u_i(s_i, S^{-i}) \quad \text{f}$$
$$\bar{S} = (\bar{s}_1, \ldots, \bar{s}_N)$$
$$u_i(\bar{s}_i, \bar{S}^{-i}) \geq$$

Examples (N =2)

Prisoner's Dilemma

one Nash equilibrium

I/II	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

OSS: A Nash equilibrium is *inefficient* when is not *pareto optimal*.

Battle of sexes

two Nash equilibria

him/her	$football$	$opera$
$football$	2,1	0,0
$opera$	0,0	1,2

Head and Tail

no Nash equilibrium

I/II	H	T
H	-1,1	1,-1
T	1,-1	-1,1

Generalization and Refinements

- Generalization: ***Mixed strategies Nash equilibrium***
- A further generalization of the Nash equilibrium concept is the **rationalizability**

Oddities in the Nash equilibrium:

- ❖ inefficiency (Prisoner's dilemma)
- ❖ non-uniqueness (Battle of sexes, Pure coordination)
- ❖ non-existence (Head and Tail)

- In order to avoid the *non-existence* and *multiple Nash equilibria:*

  I. ***Correlated equilibrium***
  II. *Perfect subgame equilibrium*
  III. *Trembling hand perfect equilibrium*

All failed w.r.t. uniqueness and *efficiency* ⟶ need to account for cooperation (Cooperative games)

Mixed strategies

- Attempt: to generalize the Nash equilibrium concept (pure strategy)
- Probabilistic approach: we each player choose a probability distribution over his set of strategies (independently) insead of choosing a single deterministic strategy

<u>Definition</u> (Mixed strategy)
A **mixed strategy** α_i for player *i* is a probability distribution over his set of strategies (actions)

m $S_i = \{s_i^1, s_i^2, ..., s_i^m\}$ $\alpha_i \in \Delta(S_i)$

- Pure strategy profile : $s = (s_1, s_2, ..., s_N)$;
- Mixed strategy profile: $\alpha = (\alpha_i)_{i \in N}$ $\alpha \in \Delta(S)$

- Given α (p.d. over deterministic outcomes), the **expected payoff** of player i is a function $U_i : \times_{j \in N} \Delta(A_j) \rightarrow \mathbb{R}$ defined as

$$U_i(\alpha) = \sum_{s \in S} (\prod_{j \in N} \alpha_j(s_j)) u_i(s)$$

i.e. the expected value of $u_i : \times_{j \in N} S_j \rightarrow \mathbb{R}$ induced by α

Mixed strategy game

<u>Definition</u>

Given α (p.d. over deterministic outcomes), the **expected payoff** of player i is a function $U_i : \times_{j \in N} \Delta(A_j) \rightarrow \mathbb{R}$ defined as

$$U_i(\alpha) = \sum_{s \in S} (\prod_{j \in N} \alpha_j(s_j)) u_i(s)$$

m i.e. the expected value of $u_i : \times_{j \in N} S_j \rightarrow \mathbb{R}$ induced by α

- The strategic game $\langle N, (\Delta(S_i)), (U_i) \rangle$ is the **mixed extension** of the strategic game $\langle N, (S_i), (u_i) \rangle$

↓

A **mixed strategies Nash equilibrium** of a strategic game is a Nash equilibrium of the mixed extension

Mixed strategy Nash equilibrium

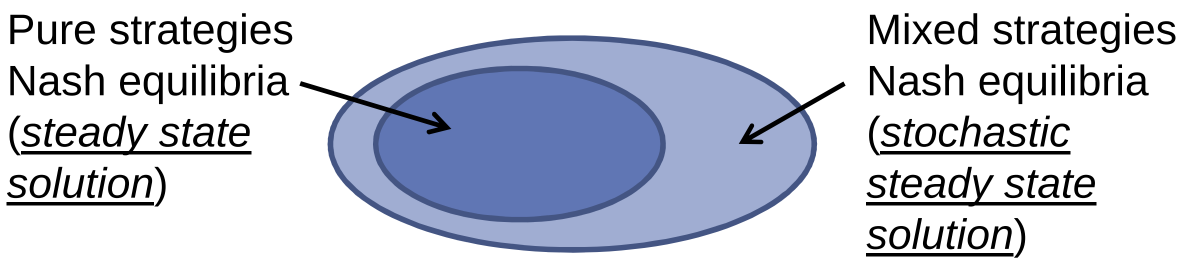
<u>Definition</u> (Mixed strategies equilibrium)

A mixed strategy profile is a **mixed strategy Nash equilibrium** if

$$U_i(\alpha^*) \geq U_i(\alpha_i, \alpha^*_{-i}) \quad \forall \alpha_i,\ \forall i \in N \qquad (\alpha^*_i \in B_i(\alpha^*_{-i}) \quad \forall i \in N)$$

<u>Properties</u>

- The set of pure strategy equilibria is a *subset* of the set of the mixed strategy equilibria



- Every finite strategic game has a mixed strategy Nash equilibrium (it solves the non-existence problem)

Example (Haid and Tail)

- *No* Nash equilibrium (in pure strategies)
- *Unique* mixed strategy Nash equilibrium: ((1/2,1/2),(1/2,1/2))

$$\alpha = (\alpha_1, \alpha_2) = ((p, 1-p), (q, 1-q))$$

		q	1- q
	I/II	H	T
p	H	-1,1	1,-1
1 - p	T	1,-1	-1,1

Player 1's best expected payoff (best responce):

$$U_1(\alpha/Head) = q \cdot 1 + (1-q) \cdot (-1) = 2q - 1$$

$$U_1(\alpha/Tail) = q \cdot (-1) + (1-q) \cdot 1 = 1 - 2q$$

$$\begin{array}{lcll} q < 1/2 & \rightarrow & U_1((0,1), \alpha_2) \geq U_1(\alpha_1, \alpha_2) & \forall \alpha_1 \\ q > 1/2 & \rightarrow & U_1((1,0), \alpha_2) \geq U_1(\alpha_1, \alpha_2) & \forall \alpha_1 \\ q = 1/2 & \rightarrow & U_1(\alpha_1, \alpha_2) \text{ costante con } \alpha_1 & \end{array} \quad \longleftrightarrow \quad B_1(\alpha_2) = \begin{cases} (0,1) & q < 1/2 \\ (p, 1-p) & q > 1/2 \\ (1,0) & q > 1/2 \end{cases}$$

Player 2's best expected payoff (best responce):

$$B_2(\alpha_1) = \begin{cases} (1,0) & p < 1/2 \\ (q, 1-q) & p = 1/2 \\ (0,1) & p > 1/2 \end{cases}$$

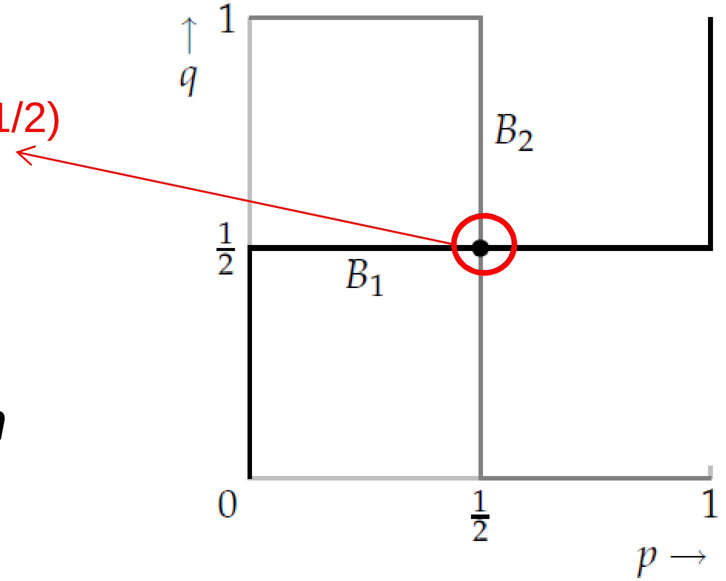
Example (Haid and Tail)

The set of mixed strategy Nash equilibria of the game corresponds to the set of *intersections of the best responce function,*

i.e. the points α^* *such that* $\alpha^* = (B_1(\alpha_2^*), B_2(\alpha_1^*))$

$$\alpha^* = ((\tfrac{1}{2}, \tfrac{1}{2}), (\tfrac{1}{2}, \tfrac{1}{2}))$$

(p*,q*) = (1/2,1/2)



The game has a unique Nash equilibrium in mixed strategies.

Example (BoS)

- Two Nash equilibria (in pure strategies)
- Tree Nash equilibria in mixed strategies

$$\alpha = (\alpha_1, \alpha_2) = ((p, 1-p), (q, 1-q))$$

		q	1- q
	him/her	*football*	*opera*
p	*football*	2,1	0,0
1 - p	*opera*	0,0	1,2

Player 1's best responce function:

$$U_1(\alpha/football) = 2 \cdot q + 0 \cdot (1-q) = 2q$$
$$U_1(\alpha/opera) = 0 \cdot q + 1 \cdot (1-q) = 1-q$$

$$\longrightarrow \quad B_1(\alpha_2) = \begin{cases} (0,1) & q < 1/3 \\ (p, 1-p) & q = 1/3 \\ (1,0) & q > 1/3 \end{cases}$$

Player 2's best responce function:

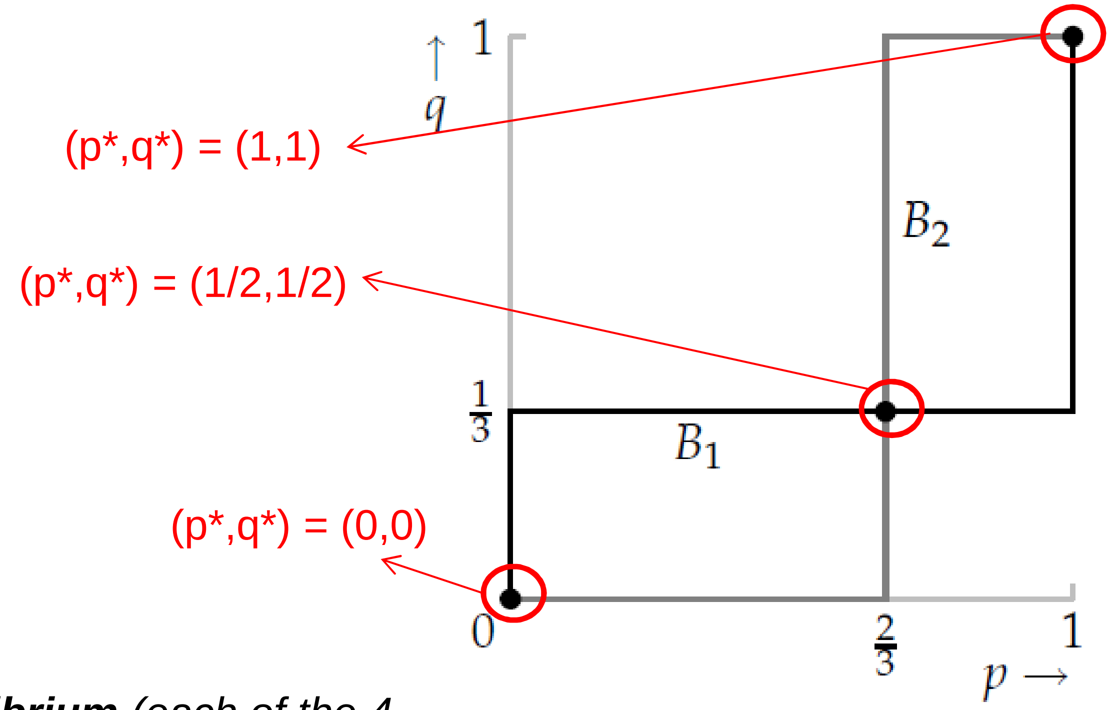
$$B_2(\alpha_1) = \begin{cases} (0,1) & p < 2/3 \\ (q, 1-q) & p = 2/3 \\ (1,0) & p > 2/3 \end{cases}$$

Example (BoS)

There are tree intersection points of the players' best responce functions.

$$\alpha^* = ((0,1),(0,1))$$
$$\alpha^* = ((1,0),(1,0))$$

are ***the Nash equilibria*** *(opera, opera)* *and* *(football,football)* ***in pure strategies***



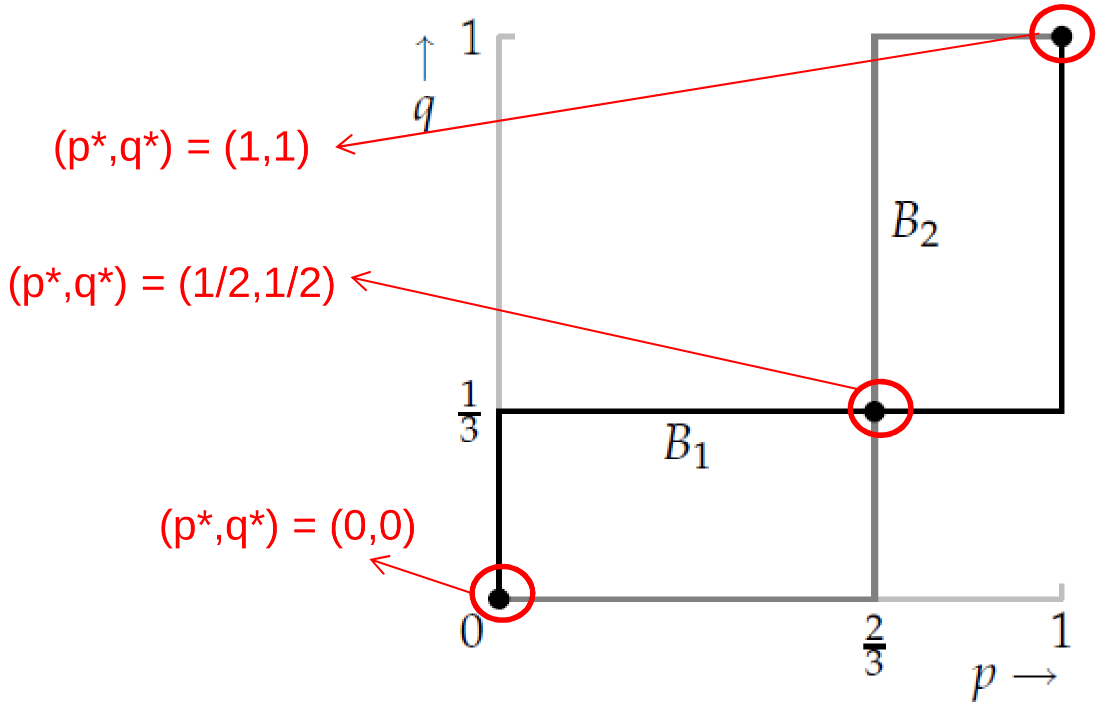
$$\alpha^* = ((\tfrac{2}{3},\tfrac{1}{3}),(\tfrac{1}{3},\tfrac{2}{3}))$$

mixed strategy Nash equilibrium *(each of the 4 deterministic outcomes occurs with positive probability)*

OSS: The mixed Nash equilibrium is *pareto dominated* by the two pure Nash equilibria.

Example (BoS)

There are tree intersection points of the players' best responce functions.



$$\alpha^* = ((0,1),(0,1))$$
$$\alpha^* = ((1,0),(1,0))$$

are ***the Nash equilibria*** *(opera, opera)* *and* *(football,football)* ***in pure strategies***

$$\alpha^* = ((\tfrac{2}{3},\tfrac{1}{3}),(\tfrac{1}{3},\tfrac{2}{3}))$$

mixed strategy Nash equilibrium *(each of the 4 deterministic outcomes occurs with positive probability)*

Rationalizability

❑Assumption: each player *knows* that the other players are *intelligent* and *rational*

- A strategy s is a **rationalizable equilibrium** if an infinite sequence of reasoning (consistent beliefs) results in the players playing s
- How to find *rationalizable strategies*?

to look for *non-rationalizable actions* and eliminate them

<u>*Def*</u> : an action of player i is a **never-best responce** if it is not a best responce to any belief of player i

Never-best responce ⟶ *non rationalizable* (see the *Prisoner's dilemma*)

<u>*Def*</u>: the strategy $s_i \in S_i$ of player i is **strictly dominated** if there exists a mixed strategy $\alpha_i \in \Delta(S_i)$ of player i that strictly dominates, i.e.

$$U_i(\alpha_i, s_{-i}) > u_i(s_i, s_{-i}) \quad \forall s_{-i} \in S_{-i} \qquad s_{-i} \in S_{-i}$$

⟶ *A strictly dominated strategy is a never best responce*

S_i: never best responce ⟺ strictly dominated

Iterated elimination of Strictly Dominated Actions

1. Eliminate strictly dominated actions from the game because no rational player plays such actions;
2. Even more actions can be strictly dominated within the remaining game; so eliminate them;
3. Further actions can be eliminated since each player is rational, believes that thge other players are rational, and belives that the other players believe that the other players are rational……
4. For a finite game, the process of successive eliminations stop at some point;

……obtaining the set of **all rationalizable strategies.**

- **Nash equilibrium** $\Rightarrow$ **Rationalizable equilibrium**

  $\not\Leftarrow$

<u>Oss</u>: the rationalizability concept looks at the game from the point of view of a *single* player

$u_i(\bar{s}_i, \bar{S}^{-i}) \geq u_i(s_i, \bar{S}^{-1})$ f

$u_i(s_i^*, S^{-i}) \geq u_i(s_i, S^{-i})$ f

$\bar{S} = (\bar{s}_1, \ldots, \bar{s}_N)$ $u_i(\bar{s}_i,$

Examples

❖ **Head and Tail**

No elimination is possible; *all the pure strategies in this game are rationalizable*

I/II	H	T
H	-1,1	1,-1
T	1,-1	-1,1

S^-

❖ The same happens in any *coordination game* (players choose corresponding strategies).

$u_i(\bar{s}$

Es: **Pure coordination game**

I/II	*football*	*rugby*
football	1,1	0,0
rugby	0,0	1,1

❖ **Prisoner's dilemma**

I/II	C	NC
C	-3,-3	0,-5
NC	-5,0	-1,-1

Rationalizable equilibrium

❖ **Typical example**

1/2	L	C	R
T	4,5	1,7	5,6
M	3,4	2,5	5,4
B	2,5	1,1	7,0

Rationalizable equilibrium

Games with communication

- *To solve «inefficiency» and «non-uniqueness» of the Nash equilibrium ;**communication among players***
- ***Communication*** $\not\rightarrow$ ***Cooperation***
- The introduction of communication among players can lead to a *Self-enforcing equilibrium (without binding agreement)*

<u>Definition</u> (Generalized strategy)

A **correlated strategy** or **jointly randomized strategy** for a set of players $C \subseteq N$ is any probability distribution α over the set of possible combinations of pure strategies these players can choose, i.e. $\alpha \in \Delta(S_c) = \Delta(\times_{i \in C}(S_i))$

Correlated strategy profile vs **Mixed strategy profile**

$\alpha \in \Delta(\times(S_i))$ $\alpha \in \times\Delta(S_i))$

In a correlated strategy the mixed strategies can be correlated

Correlated strategies and equilibrium

A *correlated strategy* α can be implemented by the players through a ***mediator*** which recommends randomly a profile of pure strategies according to α

Correlated equilibrium (Aumnann, 1974)

«Any correlated strategies for the players which could be self-enforcingly implemented with the help of a mediator who makes non binding recommendations to each player»

- Refinement of the mixed Nash equilibrium
- Includes communication among players (public signal/ recommended strategy)

Correlated equilibrium

Definition

The **expected payoff** to player *i* when a correlated strategy $\alpha \in \Delta(S)$ is implemented is $U_i(\alpha) = \sum_{s \in S} \alpha(s) u_i(s)$

Mediator suggestion: $\alpha^* \in \Delta(S)$

- $\delta_i : S_i \to S_i$, *for each player i* ($\delta_i(s_i) = s_i$ means that player i obeys the mediator)

Definition (Correlated equilibrium)

The correlated strategy α^* induce an **equilibrium** for all players to obey the mediator recommendation if and only if

$$U_i(\alpha) = \sum_{s \in S} \alpha(s_i, s_{-i}) u_i(s_i, s_{-i}) \geq \sum_{s \in S} \alpha(s_i, s_{-i}) u_i(\delta_i(s_i), s_{-i})$$

$\forall i \in N, \forall \delta_i : S_i \to S_i$

An example

unfair

Payoff allocation of pure Nash equilibria: (5,1), (1,5)

mixed Nash equilibrium (2.5,2.5)

inefficient

1/2	x_2	y_2
x_1	5,1	0,0
y_1	4,4	1,5

Drawback: ‘non-uniqueness’ and ‘inefficiency’

- A better outcome than (2.5, 2.5) can be obtained through correlated strategies
  - es: $\alpha(x_1, x_2) = \alpha(y_1, y_2) = \frac{1}{2}; \quad \alpha(x_1, y_2) = \alpha(y_1, x_2) = 0$

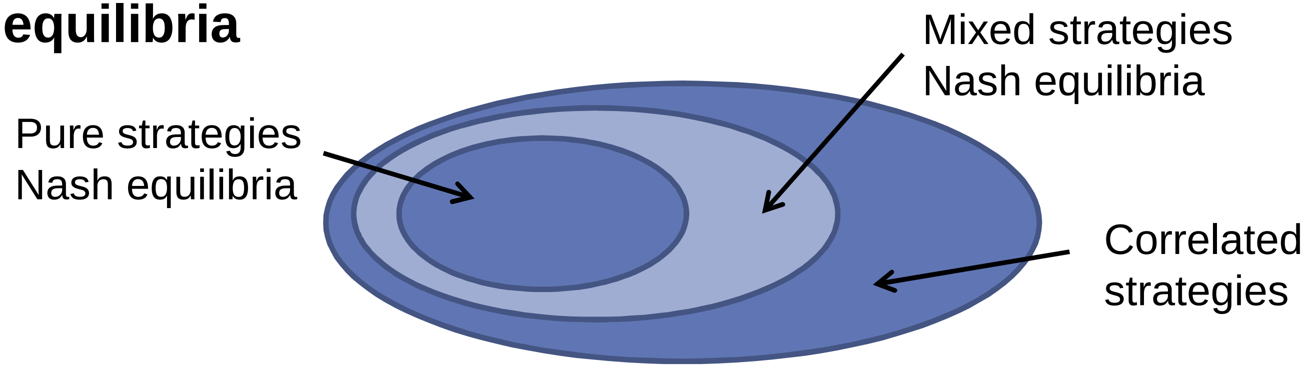
  is a *self-enforcing plan* with expected payoff (3,3)

  - es: $\alpha(x_1, x_2) = \alpha(y_1, x_2) = \alpha(y_1, y_2) = \frac{1}{3}; \quad \alpha(x_1, y_2) = 0$

  is a *self-enforcing plan* with expected payoff (3 + 1/3, 3 + 1/3)

Properties of Correlated equilibria

- The **set of correlated equilibria** contains the **set of mixed strategies Nash equilibria**



- The set of correlated equilibria includes outcomes which are **Pareto efficient** (not Pareto dominated by the pure Nash equilibria)
- Finding correlated equilibria is *computationally less expensive* than searching for Nash equilibria (*LP problem*)

Linear programming problem (LPP)

- The set of correlated equilibria is a *compact* and *convex* set
- Finding **the correlated equilibrium that maximize the sum of the player's expected payoff** is equivalent to solve the following LPP

$$\max_{i \in N} \sum_{i \in N} U_i(\alpha)$$

$$\begin{aligned} \sum_{s_{-i} \in S_{-i}} \alpha(s)[u_i(s_i, s_{-i}) - u_i(s_i', s_{-i})] &\geq 0 \quad \forall i \in N, \forall s_i \in S_i, \forall s_i' \in S_i \\ \alpha(s) &\geq 0 \quad \forall s \in S \\ \sum_{s \in S} \alpha(s) &= 1. \end{aligned}$$

By solving the linear problem in the previous example, among all the correlated equilibria $\alpha = (\frac{1}{3}, 0, \frac{1}{3}, \frac{1}{3})$ is the 'best' one.

STRATEGIC GAMES

Games with Imperfect Information

Bayesian Games: an example (1)

Bayesian Games = **Games with Imperfect Information in strategic form**

<u>*Example*</u> *(Variant of BoS with imperfect information)*

1

prob. $\frac{1}{2}$ — 2

	B	S
B	2, 1	0, 0
S	0, 0	1, 2

2 wishes to meet 1

prob. $\frac{1}{2}$ — 2

	B	S
B	2, 0	0, 2
S	0, 1	1, 0

2 wishes to avoid 1

- two ***states*** with different Player's preferences;
- from player 1's point of view Player 2 has two ***types****;*
- Player 1 has **beliefs** about the type of Player 2 (coming from experience or updated as the play takes place): ½ and ½

Bayesian Games: an example (2)

- Expected payoffs of Player 1 for the possible pairs of strategies of the two types of Player 2

	(B,B)	(B,S)	(S,B)	(S,S)
B	2	1	1	0
S	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Pure strategy Nash equilibrium = triple of strategies (one for P1 and one for each type of P2) with the property that

- ✓ the strategy of P1 is *optimal*, given the actions of the two types of P2 (and P1's belief about the state)
- ✓ the action of each type of P2 is *optimal*, given the action of P1

(B,(B,S)) is a Nash equilibrium

The *types* must be treated as separate *players!*

Bayesian Games

A Bayesian game consists of:

- a set of **players** N
- a set of **states** $\omega \in \Omega$
- a set of **strategies** S_i for each player *i*
- a finite set T_i of **types** of player i and a function $\tau_i : \Omega \to t_i$ which assigns a type to any state for player *i*
- a probability measure p_i on Ω for each player *i* (the **prior belief** of *i*)
- Bernoulli payoffs $u_i : S \times \Omega \to \mathbb{R}$ for each player *i*

Definition

A **Nash equilibrium of a Bayesian Game** is *a Nash equilibrium of the strategic game* defined as follows

- the set of players $(i, t_i), \quad i \in N \; t_i \in T_i$
- the set of strategies $S_{(i,t_i)}$ for each player (i, t_i), $S = \times S_{(i,t_i)}$
- the Bernoulli payoffs $u_{(i,t_i)} : S \to \mathbb{R}$ for each player (i, t_i) is the expected payoff of type t_i of player *i*

Bayesian Games

A Bayesian game consists of:

- a set of **players** N
- a set of **states** $\omega \in \Omega$
- a set of **strategies** S_i for each player *i*
- a finite set T_i of **types** of player i and a function $\tau_i : \Omega \to t_i$ which assigns a type to any state for player *i*
- a probability measure p_i on Ω for each player *i* (the **prior belief** of *i*)
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<u>*Definition*</u>

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